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Decision making under uncertainty in energy systems: State of the art

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ABSTRACT

The energy system studies include a wide range of issues from short term (e.g. real-time, hourly, daily and weekly operating decisions) to long term horizons (e.g. planning or policy making). The decision making chain is fed by input parameters which are usually subject to uncertainties. The art of dealing with uncertainties has been developed in various directions and has recently become a focal point of interest. In this paper, a new standard classification of uncertainty modeling techniques for decision making process is proposed. These methods are introduced and compared along with demonstrating their strengths and weaknesses. The promising lines of future researches are explored in the shadow of a comprehensive overview of the past and present applications. The possibility of using the novel concept of Z-numbers is introduced for the first time.

Contents

1. Introduction

The uncertainty handling has been one of the main concerns of the decision makers (including governors, engineers, managers, and scientists) for many years [1]. Most of the decisions to be made by energy sector decision makers are subject to a significant level of

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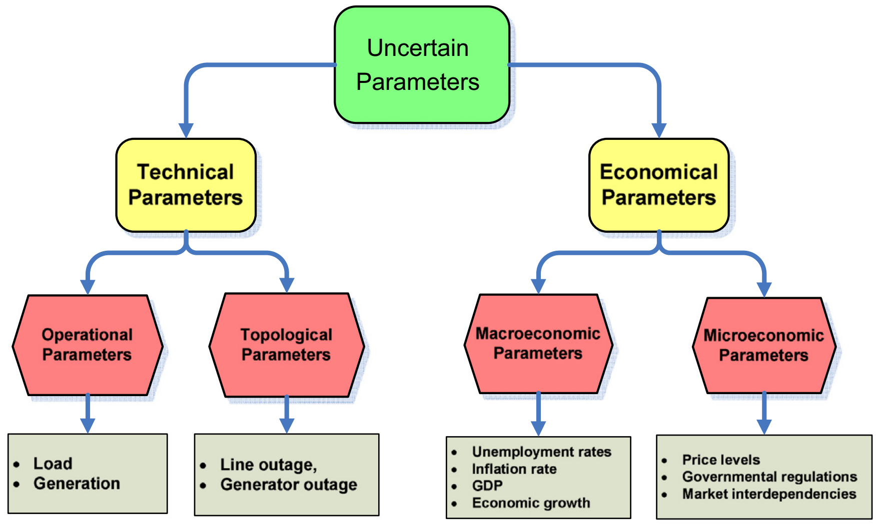


Fig. 1. General classification of uncertain parameters in energy system studies.

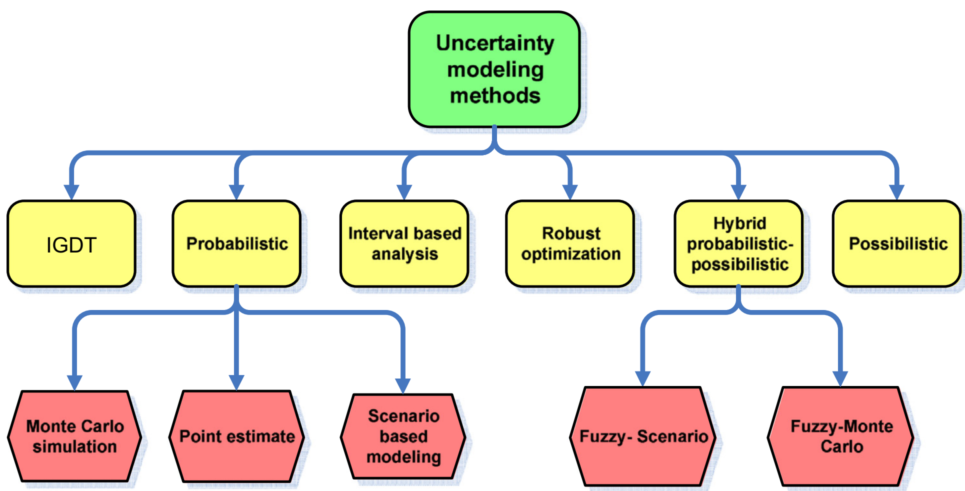


Fig. 2. Uncertainty modeling tools.

data uncertainty [2]. The uncertain parameters in power system studies can be generally classified into two different categories including (see Fig. 1):

- Technical parameters: These parameters are generally categorized in two main classes, namely topological parameters and operational parameters. The topological parameters are those related to network topologies like failure or forced outage of lines, generators or metering devices, etc. The operational parameters are tied with operating decisions like demand or generation values in power systems.
- Economical parameters: The parameters which affect the economical indices fall in this category. Microeconomics investigates the decisions of smaller business sectors like aggregators, domestic or industrial consumers while macroeconomics focuses on the entire power system industry. For example, uncertainty in fuel supply, costs of production, business taxes, labor and raw materials are analyzed in microeconomics. On the other hand, the issues like regulation or deregulation, environmental policies, economic growth, unemployment rates, gross domestic product (GDP) and interest rates are analyzed in macroeconomics. All of these parameters are subject to uncertainties and should be correctly addressed in economical studies.

There are various uncertainty handling methods developed for dealing with the aforementioned uncertain parameters as depicted in Fig. 2. The main difference between these methods is in line with the different techniques used for describing the uncertainty of input parameters. For example, fuzzy method uses membership functions for describing an uncertain parameter while the stochastic methods use probability density function. The similarity of them is that all of them try to quantify the effect of input parameters on model's outputs. These methods and the way the uncertainty is handled by them are described as follows:

- Probabilistic approach: One of the earliest works in stochastic programming was done by Dantzig in 1955 [3]. It is assumed that the input parameters of the model are random variables with a known probability density function (PDF).
- Possibilistic approach: The fuzzy arithmetic was introduced by Lotfi A. Zadeh in 1965 [4]. The input parameters of the model are described using the membership function (MF) of input parameters.
- Hybrid possibilistic–probabilistic approaches: Both random and possibilistic parameters are present in the model.
- Information gap decision theory: It was first proposed by Yakov Ben-Haim [5] in 1980. In this method, no PDF or membership function is available for input parameters. It is based on the

difference between what is known and what is vital to be known by quantification of severe lack of information in the decision making process.

- Robust optimization: It was first proposed by Soyster [6] in 1973. The uncertainty sets are used for describing the uncertainty of input parameters. Using this technique, the obtained decisions remain optimal for the worst-case realization of the uncertain parameter within a given set.
- Interval analysis: It was introduced by Ramon E. Moore in 1966 [7]. It is assumed that the uncertain parameters are taking value from a known interval. It is somehow similar to the probabilistic modeling with a uniform PDF. This method finds the bounds of output variables.

This paper is to provide a summary of recent techniques used for uncertainty modeling in power system applications. It offers a vision obtained from a relatively large number of previous works. This review serves as a road map to those interested in uncertainty modeling tools in power system studies to find the less explored research areas by standing on the shoulders of giants.

The rest of this paper is set out as follows: Section 2 presents the Probabilistic approach, the possibilistic methodology is introduced in Section 3, the hybrid possibilistic–probabilistic approach is described in Section 4, the info-gap decision theory is explained in Section 5, the robust optimization technique is described in Section 6. Section 7 presents the interval analysis approach. Section 10 describes the promising lines of future researches. Finally, Section 11 summarizes the findings of this work.

2. Probabilistic approach

In the probabilistic approach, a multivariate function, namely y, $y=f(Z)$ is available. Z is a vector of the form $Z=[z_1,\ldots,z_m]$, in which z_1 to z_m are random parameters with known PDFs while the PDF of y is tried to be identified. For a better explanation, the function f describes the system model (e.g. set of load flow equations), Z is a vector of input uncertain parameters to the system (e.g. power injections by renewable energy resources and electric loads) and y is the output variable (e.g. total active losses, total operating costs). Three probabilistic uncertainty modeling techniques are described as follows:

2.1. Monte Carlo simulation (MCS)

The Monte Carlo simulation is carried out in the following steps [8]. It is assumed that the z_i are uncertain parameters. A sample, z_i^e, is generated for each input parameter z_i, using its PDF. The value of y^e as the outcome variable, is calculated using $y^e=f(Z^e)$ where $Z^e=[z_1^e,\ldots,z_m^e]$. The procedure is repeated for a number of iterations, N_{MC}. Finally, the outcomes are analyzed using statistic criteria, histograms, confidence intervals, etc. There are some methods for reducing the computational burden of MCS like Latin Hypercube Sampling (LHS) [9], sample-splitting approach [10] and fission and roulette method [11].

2.2. Point estimate method

The point estimate method (PEM) acts based on the concept of moments of uncertain input parameters. In a problem with n uncertain parameters, the major steps are as follows [12]:

*Step*1: Set $E(Y)=0, E(Y^2)=0$ and $k=1$.

*Step*2: Determine the locations and probabilities of concentrations, $\epsilon_{k,i}$ and $P_{k,i}$, respectively as follows:

$$\epsilon_{k,i}=\frac{1}{2}\frac{M_3(z_k)}{\sigma_{z_k}^3}+(-1)^{i+1}\sqrt{n+\frac{1}{2}\left(\frac{M_3(z_k)}{\sigma_{z_k}^3}\right)^2} \tag{1}$$

$$P_{k,i}=(-1)^i\frac{\epsilon_{k,3-i}}{2n\sqrt{n+\frac{1}{2}\left(\frac{M_3(z_k)}{\sigma_{z_k}^3}\right)^2}} \tag{2}$$

where $M_3(z_k)$ is the third moment of parameter z_k.

*Step*3: Determine the concentration points $z_{k,i}$, as given below:

$$z_{k,i}=\mu_{z_k}+\epsilon_{k,i}\times\sigma_{z_i},\quad i=1,2 \tag{3}$$

where, μ_{z_k} and σ_{z_k} are mean and standard deviation of z_k, respectively.

*Step*4: Calculate the f for both $z_{k,i}$, as

$$Z=[z_1,z_2,\ldots,z_{k,i},\ldots,z_n],\quad i=1,2 \tag{4}$$

*Step*5: Calculate $E(Y)$ and $E(Y^2)$ using:

$$E(Y)=E(Y)+\sum_{i=1}^{2}P_{k,i}f(z_1,z_2,\ldots,z_{k,i},\ldots,z_n) \tag{5}$$

$$E(Y^2)=E(Y^2)+\sum_{i=1}^{2}P_{k,i}f^2(z_1,z_2,\ldots,z_{k,i},\ldots,z_n) \tag{6}$$

*Step*6: $k=k+1$ if $k<n$ then go to Step 2; otherwise continue.

*Step*7: Calculate the mean and the standard deviation as

$$\mu_Y=E(Y) \tag{7}$$

$$\sigma_Y=\sqrt{E(Y^2)-E^2(Y)} \tag{8}$$

2.3. Scenario based decision making

A scenario is defined as a probable realization of an uncertain set of parameters. A list of scenarios is generated using the PDF of each uncertain parameter, Z_s. The expected value of output variable, y, is calculated as follows:

$$y=\sum_{s\in\Omega_J}\pi_s\times f(Z_s) \tag{9}$$

where $\sum_{s\in\Omega_J}\pi_s=1$ and π_s is the probability of s^{th} scenario.

If the number of scenarios are large then it is needed to obtain a small set of scenarios representing the original one. The purpose is to select a small set, Ω_S, with the cardinality of N_{Ω_S}, from the original set, Ω_J [13]. A reasonable trade off must be respected between the loss of the information and decreasing the computational burden [2]. The scenario reduction technique is carried out via the following steps [14,15]:

*Step*1: Construct the probability distance matrix containing the distance between each pair of scenarios $c(s,s)$

*Step*2: Select the fist scenario s_1 as follows:

$$s_1=arg\left\{\min_{s'\in\Omega_J}\sum_{s\in\Omega_J}\pi_s c(s,s')\right\} \tag{10}$$

$$\Omega_S=\{s_1\},\quad \Omega_J=\Omega_J-\Omega_S \tag{11}$$

*Step*3: Select the next scenario for Ω_S set, as follows:

$$s_n=arg\left\{\min_{s'\in\Omega_J}\sum_{s\in\Omega_J-\{s'\}}\pi_s\min_{s''\in\Omega_S\cup\{s\}}c(s,s'')\right\} \tag{12}$$

$$\Omega_S = \Omega_S \cup \{s_n\}, \Omega_J = \Omega_J - \Omega_S \tag{13}$$

*Step*4: If the cardinality of Ω_S is sufficient then go to step 2; else continue.
*Step*5: Add the probability of each non-selected scenario to its closest scenario in the selected set, End.

More detail can be found in [2].

3. Possibilistic approach

Since the introduction of fuzzy set theory this technique has been used in many power system fields [16]. Suppose $y = f(x_1, \ldots, x_n)$ is in hand and X vector contains the uncertain input parameters described using their associated membership functions. In this context, the function f describes the system model (e.g. self-scheduling problem for a genco in a liberalized electricity market), X is a vector of input uncertain parameters to the system (e.g. hourly electricity price) and y is the output variable (e.g. total profit of genco).

Various membership functions can be used to formulate the degree of membership of a specific uncertain parameter depending on the expert's opinion. Regardless of the membership function's shape the question is "how to determine the MF of y if MFs of X are known?". The $\alpha-$cut method can provide an answer to this question [17]. For a given fuzzy set $\tilde{A}$ in U, the crisp set A^α contains all individuals of U with membership value, $\tilde{A}$, greater than or equal to α, as calculated in

$$A^\alpha = \{x \in U | \mu_A(x) \geq \alpha\} \tag{14}$$

$$A^\alpha = (\underline{A}^\alpha, \overline{A}^\alpha) \tag{15}$$

The $\alpha-$cut of each uncertain parameter, x_i^α, is determined using (14), then the $\alpha-$cut of y, y^α, is calculated as follows:

$$y^\alpha = (\underline{y}^\alpha, \overline{y}^\alpha) \tag{16}$$

$$y^\alpha = (\min_{X^\alpha} f(X^\alpha), \max_{X^\alpha} f(X^\alpha)) \tag{17}$$

$$X^\alpha = (\underline{X}^\alpha, \overline{X}^\alpha) \tag{18}$$

In each $\alpha-$cut, the upper bound of y^α, $\overline{y}^\alpha$, and the lower bound of y^α, $\underline{y}^\alpha$, are maximized and minimized respectively. The final step is defuzzification. The process of translating a fuzzy number to a crisp one is called defuzzification [17]. Many defuzzification techniques are available such as maximum defuzzification technique, the centroid method [18], weighted average defuzzification technique.

4. Hybrid possibilistic–probabilistic approach

Sometimes, the decision maker is faced with a multivariate objective function, $y = f(X, Z)$, where both possibilistic uncertain parameters (X) and probabilistic uncertain ones (Z) exist.

For better clarification, the function f describes the system model (e.g. set of load flow equations), Z is a vector of input uncertain parameters to the system described by PDF (e.g. power injections by renewable energy resources and electric loads), X is a vector of input uncertain parameters to the system described by MF (e.g. electricity prices) and y is the output variable (e.g. total payments for procurement of active power losses).

To deal with such cases some methods are developed which are described next.

4.1. Possibilistic-Monte Carlo approach

The following steps describe the mixed possibilistic-Monte Carlo approach [19]:

*Step*1: For each $z_i \in Z$, generate a value using its PDF, z_i^e.
*Step*2: Calculate $(\overline{y}^\alpha)^e$ and $(\underline{y}^\alpha)^e$ as follows:

$$(\underline{y}^\alpha)^e = \min f(Z^e, X^\alpha) \tag{19}$$

$$(\overline{y}^\alpha)^e = \max f(Z^e, X^\alpha) \tag{20}$$

$$X^\alpha = (\underline{X}^\alpha, \overline{X}^\alpha) \tag{21}$$

These steps are repeated to obtain the statistical data of the parameters of the output's MF such as PDF or expected values.

4.2. Possibilistic-scenario based approach

The following steps describe this approach [20]:

*Step*1: Generate the scenario set describing the behavior of Z, Ω_J.
*Step*2: Reduce the original scenario set to a small set, Ω_s.
*Step*3: Calculate $(\overline{y}^\alpha)$ and $(\underline{y}^\alpha)$ as follows:

$$\underline{y}^\alpha = \min \sum_{s \in \Omega_s} \pi_s \times f(Z_s, X^\alpha) \tag{22}$$

$$\overline{y}^\alpha = \max \sum_{s \in \Omega_s} \pi_s \times f(Z_s, X^\alpha) \tag{23}$$

$$X^\alpha = (\underline{X}^\alpha, \overline{X}^\alpha) \tag{24}$$

*Step*4: Deffuzzify the y.

5. Information gap decision theory

The Information Gap Decision Theory (IGDT) is a method to describe the uncertainties which cannot be described using PDF of MF due to the lack of sufficient information. It is used to make robust decisions against severe uncertainty of input parameters. Consider a typical optimization function as follows:

$$y = \min_d f(X, d) \tag{25}$$

$$H(X, d) = 0 \tag{26}$$

$$G(X, d) \geq 0 \tag{27}$$

where X is the vector of input parameters (which are subject to severe uncertainty) and d is the vector of decision variables. H and G are the equality and inequality constraints, respectively. $f(X, d)$ describes the relations between the decision variables (d) and input uncertain parameters (X).

In case the uncertain input parameters X are equal to their predicted values ($X = \overline{X}$) then solving the (25)–(27) gives the predicted value of $y = \overline{y}$. However, if the value of X is unknown then the IGDT method tries to find a solution for the problem which is robust against the error in predicting the value of X. In IGDT, the robustness is defined as the immunity of satisfaction of a predefined constraint [5]. The constraint satisfaction may be defined based on the application [21].

For a better clarification, assume that the function f describes the system model (e.g. set of constraints describing energy procurement from different resources), X is a vector of input uncertain parameters to the system which are subject to severe uncertainty (e.g. electricity price without any historic data) and y

is the output variable (e.g. total payments for energy procurement). d denotes the set of decision variables (e.g. amount of purchased energy from different energy resources like DG units, electricity pool market and bilateral contracts). The robustness in IGDT context is defined as follows:

The total payments should be always less than a pre-specified threshold ℓ_c, no matter how the uncertain electricity price takes value far from what is predicted. The robust counterpart of the problem described in (25)–(27) is as follows:

$$f(X,d) \le \ell_c \quad (28)$$

$$\ell_c = (1+\zeta) \times \overline{y} \quad (29)$$

$$H(X,d) = 0 \quad (30)$$

$$G(X,d) \ge 0 \quad (31)$$

where ζ is the degree that decision maker tolerates the deterioration of objective function due to forecasting error of input parameter X. The uncertainty of parameters in IGDT method is usually defined using the envelope bound model [22], as follows:

$$\tilde{X} \in U(\alpha, \overline{X}) \quad (32)$$

$$U(\alpha, \overline{X}) = \left| \frac{X-\overline{X}}{\overline{X}} \right| \le \alpha \quad (33)$$

where α is the uncertainty level of parameter X, $\overline{X}$ is the forecasted value of X and $U(\alpha, \overline{X})$ is the set of all values of X whose deviation from $\overline{X}$ will never be more than $\alpha\overline{X}$. The decision maker does not know the values of X and α.

The robustness of a decision d based on the requirement ℓ_c, $\hat{\alpha}(d,\ell_c)$, is defined as the maximum value of α at which the decision maker is sure that the required constraints are always satisfied [5], as follows:

$$\hat{\alpha}(d,\ell_c) = \max \alpha \quad \text{S.t}: \text{ Constraints} \quad (34)$$

The decision making policy is defined as finding the decision variables, d, which maximizes the robustness, as

$$\max_d \hat{\alpha}(d,\ell_c) \quad (35)$$

$$\forall X \in U(\alpha, \overline{X}) \quad (36)$$

$$f(X,d) \le \ell_c \quad (37)$$

$$\ell_c = (1+\zeta) \times \overline{y} \quad (38)$$

$$H(X,d) = 0 \quad (39)$$

$$G(X,d) \ge 0 \quad (40)$$

6. Robust optimization

The concept of robust optimization (RO) was first introduced by Soyster [6]. It is a new approach for solving optimization problems affected by uncertainty specially in the case of lack of full information on the nature of uncertainty [23]. It is described as follows: consider a function like $z = f(X,y)$ which is linear in X and non-linear in y. The values of X are subject to uncertainty while the values of y are known. In robust optimization, it is assumed that no specified PDF is in hand for describing the uncertain parameter X. The uncertainty of X is modeled with an uncertainty set $X \in U(X)$, where $U(X)$ is a set that parameter X can take value from it. The maximization of $z = f(X,y)$ can be formulated via the following:

$$\max_y z = f(X,y) \quad (41)$$

$$X \in U(X) \quad (42)$$

Since the value of z is linear with respect to X, it can be reformulated as follows:

$$\max_y z \quad (43)$$

$$z \le f(\tilde{X}, y) \quad (44)$$

$$f(\tilde{X}, y) = A(y) * \tilde{X} + g(y) \quad (45)$$

$$\tilde{X} \in U(X) = \{X \,||X-\overline{X}| \le \hat{X}\} \quad (46)$$

where $\tilde{X}, \overline{X}, \hat{X}$ are the uncertain value, predicted value and maximum possible deviation of variable X from $\overline{X}$, respectively. The robust optimization seeks a solution which not only maximizes the objective function z but also insures the decision maker that if there exist some prediction error about the values of X, the z remains optimum with high probability [24]. To do this, a *robust counterpart* version of the problem is constructed and solved. The robust counterpart of (42) is defined as follows:

$$\max_y z \quad (47)$$

$$z \le f(X,y) \quad (48)$$

$$f(X,y) = A(y) * \overline{X} + g(y) - \max_{w_i} \sum_i a_i(y) * \hat{X}_i * w_i \quad (49)$$

$$\sum_i w_i \le \Gamma \quad (50)$$

$$0 \le w_i \le 1 \quad (51)$$

Based on (47), two nested optimization problems are to be solved. Eqs. (49) and (50) are linear with respect to w_i and has a dual form as follows:

$$\min_{\xi_i,\beta} [\Gamma\beta + \sum_i \xi_i] \quad \beta + \xi_i \ge a_i(y) * \hat{x}_i \quad (52)$$

Inserting the (52) into (47) gives

$$\max_{y,\xi_i,\beta} z \quad (53)$$

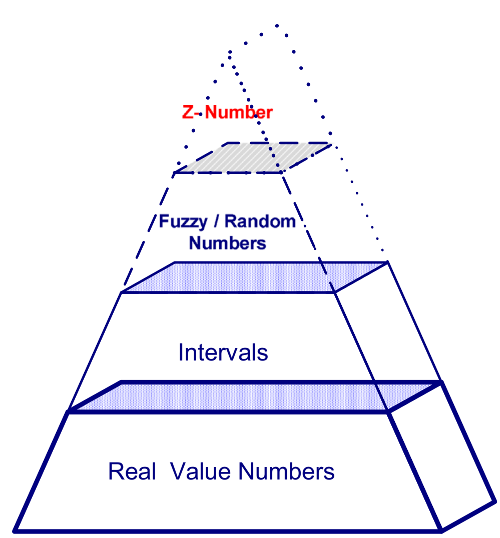


Fig. 3. Uncertainty modeling trends: past, present and future.

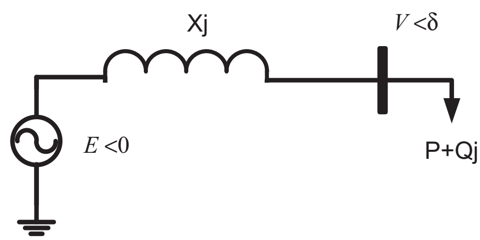


Fig. 4. Simple two-bus illustrative network.

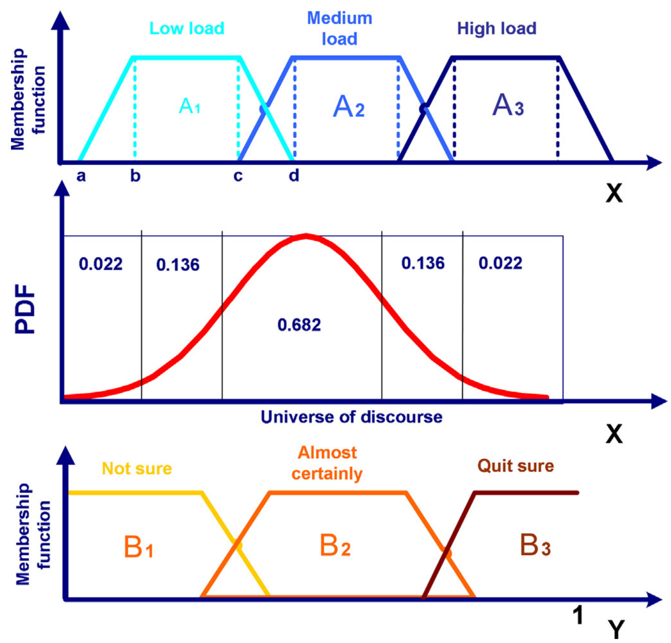


Fig. 5. Concept of Z-number.

$$z \le f(X, y) \tag{54}$$

$$f(X, y) = A(y) * \overline{X} + g(y) - \Gamma\beta - \sum_i \xi_i \tag{55}$$

$$\beta + \xi_i \ge A(y_i) * \hat{X}_i \tag{56}$$

There are some software developed for solving the robust optimization based problems [25]. As an illustrative example, consider that the function f describes the system model (e.g. set of constraints describing energy purchased by a smart home), X is a vector of input uncertain parameters to the system (e.g. electricity price which are always within a band), z is the output variable (e.g. total payments for energy procurement). Γ denotes the degree of conservativeness and y is the set of decision variables (e.g. amount of purchased energy in different hours).

7. Interval analysis

In this method, the range of values for each uncertain input parameter is defined and it can be represented by an interval. Suppose a multivariate function of the form $f = (x_1, \ldots, x_n)$ and $lb_i \le x_i \le ub_i$ where lb_i and ub_i are the lower and upper bounds of uncertain parameter x_i, respectively. The goal is finding the lower and upper bounds of the objective function f. There are some softwares developed for solving the interval analysis based problems [26]:

$$Prob = \int_a^d A_1 \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$
$$= \frac{1}{\sigma\sqrt{2\pi}} \left[\int_a^b \frac{x-a}{b-a} e^{\frac{-(x-\mu)^2}{2\sigma^2}} + \int_b^c e^{-(x-\mu)^2/2\sigma^2} + \int_c^d \frac{x-d}{c-d} e^{-(x-\mu)^2/2\sigma^2} \right] \tag{57}$$

$$G(Prob) = \mu_{B_2}(Prob) \tag{58}$$

8. Exploring the new uncertainty handling methods

The taxonomy of the uncertainty modeling methods in the past, present and future is as depicted in Fig. 3. In 2011, Zadeh introduced a new class of uncertain numbers called "*Z-numbers*" [27]. The Z-numbers are expressed as a pair in the form of $Z = (A, B)$, in which, A, B are restrictions describing the behavior of Z. A is usually a fuzzy set while B describes the certainty degree. The certainty degree may be expressed as a PDF or a fuzzy set. In this context, $Z = \{x | x \in A$ with certainty degree equal to $B\}$. In classic fuzzy numbers decision maker just has A and it is quit sure that Z belongs to A. However in Z-numbers, Z is described using the set A with a certainty (reliability) degree of information called B. Examples for Z-numbers are provided in Table 3.

The normal PDF, as a function of μ, σ, is a reasonable choice for modeling the randomness of the load variable. In order to disambiguate this concept, a simple two-bus network is used as shown in Fig. 4. The series reactance of the transmission line connecting these two buses is assumed to be X. The voltage magnitudes of sending and receiving ends are represented by E and V, respectively. The angle by which the sending end voltage leads the receiving end voltage is considered to be δ. P and Q represent the active and reactive power load at the receiving end, respectively. The parameters of Z-numbers (pairs of MF) describing load values are given in Table 4.

For example, we are *almost certain* (set B_2) that *the demand value* in a given bus (Z number) is *low* (set A_1) as depicted in Fig. 5. The probability that *the load value* is *low* can be calculated as (58).

In (58), $G(Prob)$ indicates the degree to which $Prob$ belongs to A_1. Now, the information of Z-number expressed as $L = (A_1, B_2)$ for load parameter is represented as a possibility distribution ($G(Prob)$) over the space of probability distributions (various values of μ, σ).

9. Applications

Context serves to demonstrate the applications of the aforementioned uncertainty modeling techniques. The applications are widely categorized into several fields, as given in Table 1. The summaries of uncertainty modeling attributes are provided in Table 2.

- Distributed Generation (DG) impact assessment.
- Plug-in hybrid electric vehicle (PHEV): (e.g. exploitation of plug in hybrid electric vehicles).
- Assessment of available transfer capability (ATC).
- Renewable energy (operation and planning) (e.g. hydro power generation management).
- Load flow/optimal power flow calculations (e.g. probabilistic load flow, fuzzy load flow).
- Reliability evaluation (e.g. reliability-oriented distribution network reconfiguration).
- Distribution network operation and planning (e.g. phase balancing, cost-benefit analysis of distribution automation).

Table 1
Summaries of uncertainty modeling applications.

Applications	Probabilistic			Possibilistic	Hybrid	Interval	RO	IGDT
	MC	PEM	Scenario					
DG units	[28,29]	[30]	[31,32]	[33]	[19,20]	[a]		
PHEV	[31,34]	[35]	[35]				[36]	
Available transfer capability (ATC)	[37]	[38]		[39]				
Renewable energy (operation and planning)	[40,41]	[42]	[43–47]	[48]			[49]	
Load flow/optimal power flow	[50]	[51]		[52–54]		[55]		
Reliability evaluation	[56,57]	[58]	[59,57]	[60]		[61,62]		
Distribution operation and planning	[29]		[63]	[64]		[65]		
Transmission/generation planning and operation/control	[66]		[67,68,15,69]	[70–73]	[71]	[74–76]	[77]	
State estimation	[78]	[79]	[80]	[81,82]	[81]	[83]		
Electricity market	[84,85]			[70]		[86]	[87–89]	[90,88]

[a] Unexplored research directions.

Table 2
Summaries of uncertainty modeling attributes.

Method	Input representation	Output attributes	Advantages	Disadvantages
Probabilistic	PDF	Statistics like expectation, variance, etc.	Easy to implement	Computationally expensive, needs a large amount of historic data, approximate result
Possibilistic	MF	MF	Converting linguistic knowledge to numerical values	Complex implementation
Hybrid	MF & PDF	Membership function with probabilistic parameters	Dealing with both uncertainty types simultaneously	Computationally expensive
IGDT	Forecasted values	Decision variables satisfying the requirements	Useful for severe uncertainties	Too conservative
Robust		*optimization*	Intervals	Controlled conservativeness
Useful when just an interval is available	Difficult to use in non-linear models			
Interval analysis	Intervals	Bounds of the outputs	Useful when just an interval is available	The correlations among intervals are neglected this would make it too conservative

Table 3
Examples for Z-numbers.

Parameter	A	B
Demand value	High	Very sure
Wind speed	Weibul PDF	Normally
Voltage magnitude	Uniform distribution in [0.951.05]	In most cases

Table 4
Describing the load values as Z-numbers.

A	B	Load
Low	Not sure	$L=(Low, Notsure)=(A_1,B_1)$
	Almost certain	$L=(Low, Almostcertain)=(A_1,B_2)$
	Quit sure	$L=(Low, Quitsure)=(A_1,B_3)$
Medium	Not sure	$L=(Medium, Notsure)=(A_2,B_1)$
	Almost certain	$L=(Medium, Almostcertain)=(A_2,B_2)$
	Quit sure	$L=(Medium, Quitsure)=(A_2,B_3)$
High	Not sure	$L=(High, Notsure)=(A_3,B_1)$
	Almost certain	$L=(High, Almostcertain)=(A_3,B_2)$
	Quit sure	$L=(High, Quitsure)=(A_3,B_3)$

- Transmission/Generation planning, operation and control: (e.g. self-scheduling of GenCos, fault location scheme, dynamic economic dispatch, maintenance scheduling, determination of pilot points for zonal voltage control, small-signal stability).
- State estimation.
- Electricity market (e.g. real time demand side management, bidding strategy, energy hub management and electricity procurement strategy).
- Risk analysis (e.g. risk measures, risk hedging strategies).

10. Promising lines of future researches

The future trends in uncertainty modeling (to be investigated and further explored) are summarized as follows:

10.1. Exploring new uncertain parameters

With the increasingly revolutionary changes in power system's regulatory framework and developing technologies the uncertainty in input data of decision making procedures is increased. These uncertain environments include financial, societal/governmental (the ongoing government policy and the future potential incentive for the renewable energy), environmental (carbon emission and global warming issue) and technical (communication and information architecture in smart grids, demand response, PHEV, energy hubs, smart building) uncertainties, risk preferences in the investment models, fuel prices and market regulations, renewable energy sources and competition among suppliers.

10.2. Enhancing the existing techniques

- Reduce the computational burden specially when applied to large scale power systems and real-time applications.
- Choosing the appropriate uncertainty handling technique.
- Hybridizing the existing techniques to better describe the uncertain environment.
- Using the heuristic methods to soften the computation procedures.

11. Conclusion

This paper proposed a standard classification of uncertainty handling methods along with the promising lines of future researches. The possibility of using Z-numbers for uncertainty modeling of load values was introduced for the first time. The assessed methodologies include probabilistic, possibilistic, hybrid methods, robust optimization, interval based analysis as well as Z-numbers. These models are compared and their strength and shortcomings are investigated. Based on the proposed comprehensive classification, it is deduced that each method is suitable for a specific type of uncertainty. The severity of uncertainty dictates choosing of the appropriate uncertainty modeling technique. Additionally, according to the carried out taxonomy of the methodologies, it was revealed that some research areas still remained untouched.

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